OPTIMIZATION OF STEPPED INELASTIC TUBES

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Abstract. The paper aims to investigate the nonlinear behaviour of inelastic circular cylindrical shells of piece-wise constant thickness and with additional supports subjected to the internal pressure and the axial dead load. The main purpose of the paper consists in the determination of different thicknesses and optimal positions of additional ring supports so that the cost function attains its minimum value, and the governing equations of the problem are satisfied. It is assumed that the material of the shells is an ideal rigid-perfectly-plastic one and obeys a piecewise linear yield condition and the associated flow law. According to the flow law the vector of strain rates is directed towards the external normal to the yield surface at the current point. A deformation-type theory of plasticity is applied in the thesis. According to this version the normality rule is formulated with respect to the vector of strain components. The paper concerns problem formulations with the cost functions, the governing equations including the equations of equilibrium, the gradientality law and the yield condition, also boundary conditions for displacements and bending moments. The case of a one-stepped shell clamped at the left edge and simply supported at the right edge is studied in greater detail. Numerical results are obtained for the case of the cost function which evaluates the mean deflection of the shell with a sandwich-type shell wall. In the paper semi-analytical and numerical methods are developed to determine the stress-strain state of inelastic tubes operating in the post-yield range.

Keywords: cylindrical shell; additional support; optimization; piecewise constant thickness.

Introduction

The reduction of the weight of structures or increasing the stiffness or load carrying capacity is the primary concern in many practical problems of structural engineering (see Farkas, Jarmai [1]). In the literature one can find many different approaches to the optimal design of structures (see Banichuk [2], [3]), Prager [4], Lellep [5; 6], Lellep, Hannus and Paltsepp [7], Lellep and Paltsepp [8]).

The most of work done in this area is dedicated to the elastic structures. However, due to the extremal conditions during the operation structural elements can exhibit inelastic deformations. That is why the plastic deformations can be taken into account when applying the optimization procedures.

The key problem in the limit analysis of rigid-plastic plates and shells is the problem of determination of the yield surface in the space of bending moments and membrane forces. This is an extra complicated problem in the case of shells made of materials obeying non-linear yield conditions in the space of principal stresses σ1 and σ2. A lot of light has been shed on this problem by Haydl and Sherbourne [9; 13].

The comparison of approximations of yield surfaces developed for circular cylindrical shells in undertaken by Robinson [14; 15].

The weight minimization under constrained deflections for annular plates and cylindrical shells made of Mises material was performed by Lellep and Majak [16; 17]. It is worthwhile to mention that in these studies a geometrically nonlinear model of a rigid-plastic structure is used.

Lellep and Sawczuk [18] developed a method for optimization of circular cylindrical shells in the range of large deflections. It was assumed in [18] that the material of the shell obeys a piecewise linear yield surface. The minimum weight design is established with the aid of the theory of optimal control.

An optimal design technique was developed for steel shells with technological constraints by Turnić et al [19]. In [20] the approach initiated by Drucker and Shield was extended for inelastic cylindrical shells. Ross [21] established the optimal designs for ring-stiffened shells.

Stepped inelastic tubes

The circular cylindrical shell with piece wise constant thickness will be treated. Let the circular tube of radius *R* and length *l* be subjected to axial dead load *N* and to internal pressure loading of intensity *P*. Let us consider the case of one-stepped shell clamped at the left edge and simply supported at the right edge in greater detail (Fig. 1).

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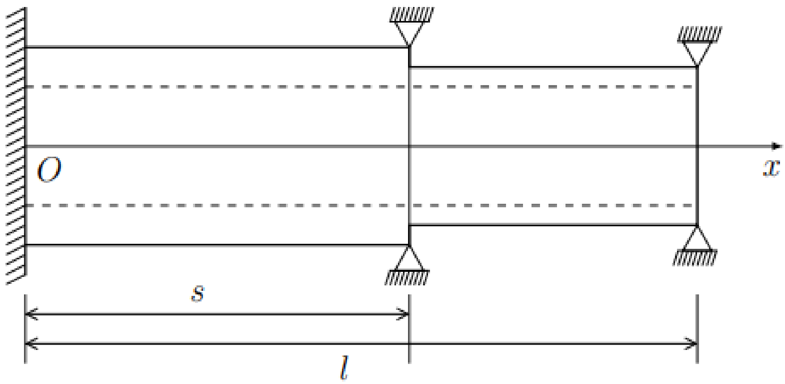

Fig. 1. **Cylindrical shell of piece wise constant thickness**

Assume that the thickness of the tube wall is defined as

$$h = \begin{cases} h_0, x \in [0, S[\\ h_0, x \in]S, l] \end{cases} \tag{1}$$

where h_0 and h_1 – given constants.

Let us treat the case of moderate high loading where both regions of the shell deform plastically.

Assume that the cost criterion is presented as (r is a positive integer)

$$J = \int_0^l W^r dx. \tag{2}$$

We are looking for the minimum of (2) so that the equilibrium equations with boundary and intermediate conditions, the associated gradientality law with the yield condition, also the inequalities

$$P - P_0 \geq 0, P - P_1 \geq 0 \tag{3}$$

are satisfied. Here P_0 and P_1 stand for the load carrying capacity of the left-hand and right-hand parts of the shell, respectively.

The material of the tube is assumed to be a perfect plastic material obeying the yield condition suggested by Lance and Robinson [22]. This condition can be considered as a piecewise linear approximation of yield conditions by Hill and Tsai-Wu (see Jones [23]) used for calculation of the failure of fibre reinforced composites.

For rotationally symmetric cylindrical shells the equilibrium equations can be presented as (Lellep [24; 25])

$$\frac{dM}{dx} = Q \tag{4}$$

and

$$\frac{dQ}{dx} = N_1 \frac{d^2W}{dx^2} - \frac{N_2}{R} + P, \tag{5}$$

where N_1, N_2 – membrane forces in the axial and hoop directions, respectively, and

$$\frac{dN_1}{dx} = 0. \tag{6}$$

The strain components corresponding to the geometrically non-linear theory consistent with equilibrium equations (4), (5) are

$$\varepsilon_1 = \frac{dU}{dx} + \frac{1}{2}\left(\frac{dW}{dx}\right)^2, \varepsilon_2 = \frac{W}{R}, \tag{7}$$

whereas the curvatures are given as

$$K_1 = \frac{d^2W}{dx}, K_2 = 0. \tag{8}$$

Boundary conditions are

$$W(0) = 0, W(S) = 0, W(l) = 0, M(0) = \frac{\alpha}{k} M_0, M(S) = \frac{\alpha}{k}\gamma M_0 \text{ and } M(0) = 0.$$

Integrating the equilibrium equations (4), (5) and satisfying appropriate boundary conditions can show that the boundaries a_0, b_0, a_1 and b_1 of the curved regions are defined as

$$\alpha_0 = \sqrt{\frac{4\alpha}{\omega k(p-k)}}l, \tag{9}$$

$$b_0 = l\left(s - \sqrt{\frac{2\alpha(1+\gamma)}{\omega k(p-k)}}\right), \tag{10}$$

$$a_1 = l\left(s + \sqrt{\frac{4\alpha\gamma}{\omega k(p-k\gamma)}}\right), \tag{11}$$

and

$$b_1 = l\left(1 - \sqrt{\frac{2\alpha\gamma}{\omega k(p-k\gamma)}}\right). \tag{12}$$

Here

$$\gamma = \frac{h_0}{h_1} \tag{13}$$

and

$$M_0 = \frac{\sigma_0 h_0^2}{4}, N_0 = 2\sigma_0 h_0.$$

Results and discussion

Transverse deflections of the shell are presented in Fig. 2 for different values of the transverse pressure p. The curves presented in Fig. 2 correspond to $p = 8$, $p = 10,\ldots, p = 18$ (here the intermediate support is located at $x = 0.54l$ and $h_1 = 0.7h_0$ or $\gamma = 0.7$).

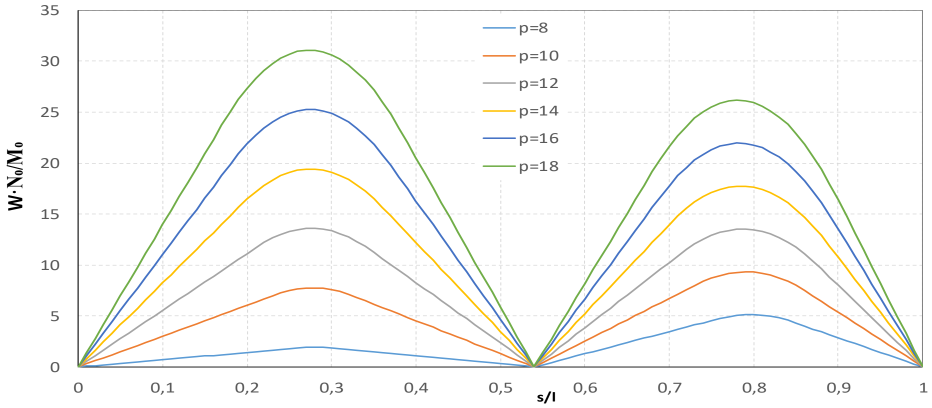

Fig. 2. **Deflections of different loadings**

It can be seen from Fig. 2 that inelastic deformations take place in the left-hand and in right-hand parts of the shell if the intensity of the pressure $p > 8$ in the present case.

If $p < 7.834$ and $p \geq p_0$ then the deformations take place in the right-hand part of the shell only.

If, for instance, $p = 18$, then the maximal deflections of the parts are $\max_{x \in (0,s)} w(x) = 31.072$ and $\max_{x \in (0,1)} w(x) = 26.190$.

The bending moments of the shell are presented in Fig. 3 for different values of the pressure p (here $S = 0.54l$, $\omega = 8$, $n_1 = 0.1$, $\gamma = 0.7$, $\alpha = k = 1.5$).

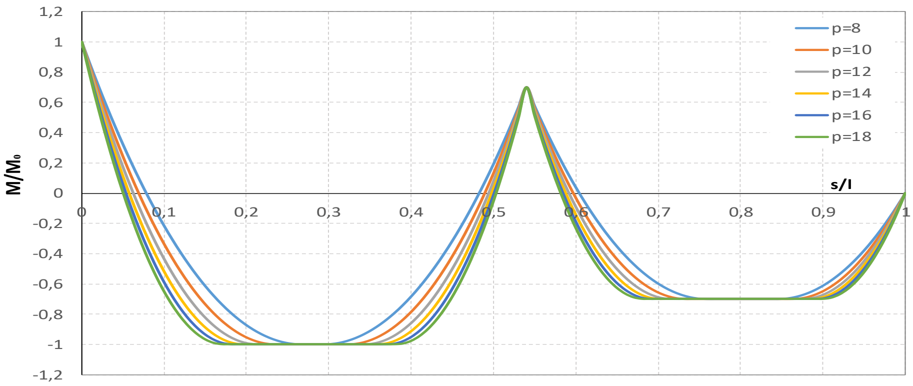

Fig. 3. **Bending moments of different loadings**

The deflections of the tube are demonstrated in Fig. 4 for different ratios of thicknesses (here $p = 12$, $S = 0.54l$).

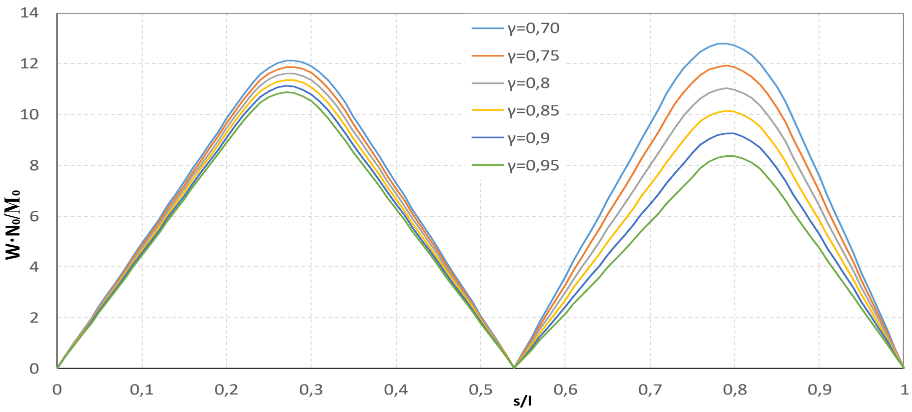

Fig. 4. **Deflections of stepped tubes for different thickness**

The bending moments corresponding to shells of piece wise constant thickness are presented in Fig. 5 for the fixed position of the additional support ($S = 0.54l$).

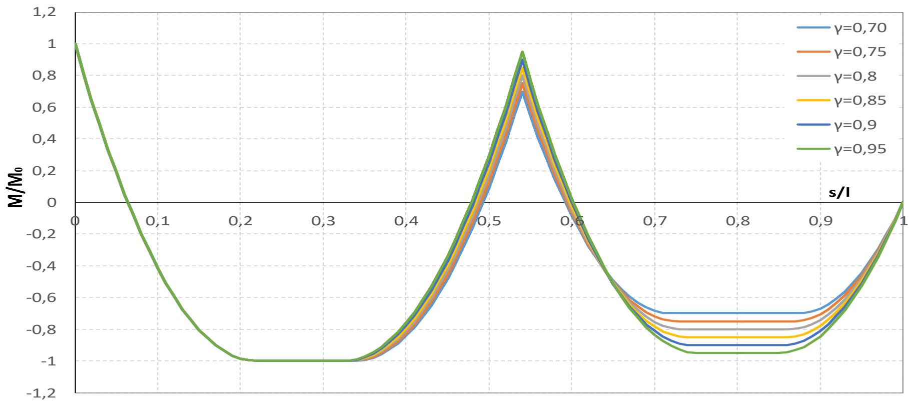

Fig. 5. **Bending moments of stepped tubes for different thickness**

The dependence between the value of the cost function and the coordinate of the additional support is shown in Fig. 6 (here = 14). It can be seen from Fig. 6 that the minimal value of the cost function is obtained if s belongs to the region (0.51l; 0.52l).

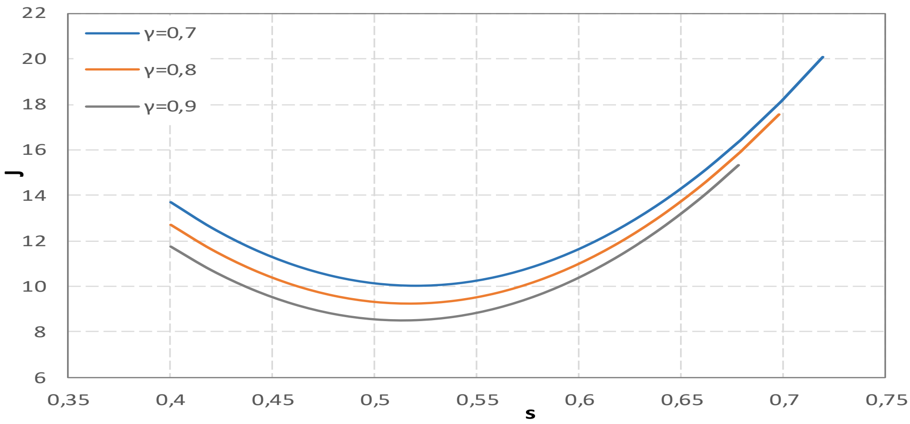

Fig. 6. **Performance index**

Conclusions

A method of optimization of locations of internal supports for a circular cylindrical shell of piece wise constant thickness was developed. The shell under consideration is clamped at the left hand end and simply supported at the right hand end. The problem is solved with the aid of Lagrange multipliers.

It appeared that the shell is not very sensitive with respect to locations of internal supports. It is shown that the transverse deflection of the shell is strongly dependent on the position of the internal support and the thickness of the wall.

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